GMR Classes

MAINS - REV TEST - 5

NOT PUBLISHED

Total Marks : 300.0
Duration : 3:00 hrs

Mathematics XI

1.

$$\lim_{x\to 0}\frac{1-\cos x-\cos 2x+\cos x.\cos 2x}{x^4}=?$$

(A) 1 (B) 2

(C) 3 (D) 4

2. Let $f(a)=g(a)=k$ and their n^{th} derivatives $f^n(a),g^n(a)$ exist and are not equal for some n. If $\lim_{x\to a}\frac{f(a)g(x)-f(a)-g(a)f(x)+g(a)}{g(x)-f(x)}=4$, then the value of k is

(A) 4 (B) 2

(C) 1 (D) 0

3.

If $f(x)=\begin{vmatrix}\cos x & x & 1\\ 2\sin x & x^2 & 2x\\ \tan x & x & 1\end{vmatrix}$, then find $\lim_{x\to 0}\frac{f'(x)}{x}=?$

(A) 1 (B) -1

(C) 2 (D) -2

4.

$$\lim_{x\to\infty}\left|\frac{1+\frac{1}{3}+\frac{1}{9}+\ldots\ldots\frac{1}{3^n}}{1+\frac{1}{5}+\frac{1}{25}+\ldots\ldots+\frac{1}{5^n}}\right|=?$$

(A) $\frac{3}{5}$ (B) $\frac{4}{5}$

(C) $\frac{6}{5}$ (D) $\frac{5}{6}$

5.

$$\lim_{x\to\infty}\left[\frac{5^{x+6}+3^{2n}-7^{x}}{8^{x}+9^{x-1}+3^{2n+1}}\right]=?$$

(A) 3 (B) $\frac{1}{3}$

(C) 9 (D) $\frac{9}{28}$

6.

$$\lim_{x\to\infty}\left(6^{n}+5^{n}\right)^{1/n}=?$$

(A) 6 (B) 5

(C) $\frac{5}{6}$ (D) e

7.

$$\lim_{x\to 1}\left(2-x\right)^{\tan\frac{\pi x}{2}}=?$$

(A) $e^{-2/\pi}$ (B) $e^{1/\pi}$

(C) $e^{2/\pi}$ (D) $e^{-1/\pi}$

8. If x = cost, y = lnt, then find the value of $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2$ at $t = \frac{\pi}{2}$

(A) 1 (B) 0

(C) π /2 (D) N one of these

9. If $x^y . y^x = e^{x^2}$ then $\frac{dy}{dx}$ at x = 1 is equal to

(A) e(1 – e) (B) $e^2 - e$

(C) $\frac{1-e}{1+e}$ (D) $\frac{1+e}{1-e}$

10. Let α, β are distinct roots of $ax^2 + bx + c = 0$, then

$$\lim_{x \to \alpha} \frac{1 - \cos\left(ax^2 + bx + c\right)}{(x-\alpha)^2} =$$

(A) $\frac{a^2}{2}(\alpha + \beta)^2$ (B) $\frac{a^2}{2}(\alpha - \beta)^2$

(C) $\frac{a^2}{2}\left(\alpha^2 - \beta^2\right)$ (D) $\frac{a^2}{4}(\alpha - \beta)^2$

11. if y = f(x) is an odd differentiable function defined on (– ∞ , ∞) such that f'(3) = – 2, then f'(–3) equals _____.

12. If $y = \sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right]$ and 0 < x < 1 then $\frac{dy}{dx}$ at $x = \frac{\sqrt{3}}{2}$ is

13.

If $f(x)=\begin{cases}\frac{|x-1|}{1-x}+a & ; \quad x>1 \\ a+b & ; \quad x=1 \\ \frac{|x-1|}{1-x}+b & ; \quad x<1\end{cases}$ is continuous at x = 1 then a and b are respectively –

(A) 1, 1

(B) 1, –1

(C) 2, 3

(D) N one of these

14.

$f(x)=\begin{cases}\frac{\sqrt{(1+px)}-\sqrt{(1-px)}}{x} & , \quad -1 \le x<0 \\ \frac{2x+1}{x-2} & , \quad 0 \le x \le 1\end{cases}$ is continuous in the interval [–1, 1], then 'p' is equal to:

(A) –1

(B) –1/2

(C) 1/2

(D) 1

15. If the curved surface of a cylinder inscribed in a sphere of radius R is a maximum, then height of the cylinder is

(A) $\sqrt{R}$

(B) $\sqrt{10R}$

(C) $\sqrt{2R}$

(D) R

16. $\text{If, then } f(x)=\begin{cases}\frac{\sin x}{x}+\cos x, & \text{when } x \neq 0 \\ 2, & \text{when } x=0\end{cases}$ then

(A) $\lim\limits_{x \to 0^+} f(x) \neq 2$

(B) $\lim\limits_{x \to 0^-} f(x)=0$

(C) $f(x)$ is continuous at $x=0$

(D) None

17. If the tangent to the curve $x = 1 - 3t^2$, $y = t - 3t^3$ at the point P (-2 , 2) meets the curve at Q then the angle between the tangents at P and Q is $\frac{\pi}{k}$, then k is

18.

If $f(x)=\begin{cases}1+x, & 0\le x\le 2\\ 3-x, & 2<x\le 3\end{cases}$ then the number of points of discontinuity of the function (fof) (x) in [0, 3] is

19. The normal to the curve x=a(1+cosθ),y=asinθ at θ always passes through the fixed point

(A) (a,a) (B) (a,0)

(C) (0,a) (D) (0,0)

20. The maximum value of $f(x) = 2x^3 - 21x^2 + 36x + 20$ in the interval $0 \le x \le 2$ is

(A) 37 (B) 38

(C) 32 (D) 30

21. The perimeter of sector is given. The area is maximum when the angle of the sector is

(A) 1 radian (B) 2 radian

(C) 3 radian (D) 4 radian

22.

The area of the triangle formed by the positive x-axis, the normal and the tangent to the curve $x^2 + y^2 = 4$ at $(1, \sqrt{3})$ in sq. Units is

(A) $2\sqrt{3}$ (B) $\sqrt{3}$

(C) $4\sqrt{3}$ (D) 6

23. The value of c in legranges mean value theorem for f(x) = log(sin x)

(A) π/4 (B) π/2

(C) 2π/3 (D) 3π/4

24.

If $f(x)=\begin{cases}3^x, & -1\le x\le 1\\ 4-x, & 1<x<4\end{cases}$, then at x = 1, f(x) will be:

(A) Continuous but not differentiable (B) Neither continuous nor differentiable

(C) Continuous and differentiable (D) Differentiable but not continuous

25.

If $f(x) = \sin^{-1} 2x\sqrt{1-x^2}$ and the values of $f'\left(\frac{1}{2}\right) + f'\left(-\frac{1}{2}\right)$ is $\frac{\lambda}{\sqrt{3}}$, then find λ.

Physics XI

26. A ball of mass 2 kg and another of mass 4 kg are dropped together from a 60 feet tall building. After a fall of 30 feet each their respective kinetic energies will be in the ratio

(A) $\sqrt{2} : 1$ (B) $2 : 1$

(C) 1:2 (D) $1 : \sqrt{2}$

27.

A pump can hoist 9000 kg of coal per hour from a mine of 120m deep. Then the power in watts, assuming that its efficiency is 75%, is

(A) 4920 watt (B) 5920 watt

(C) 4000 watt (D) N one of these

28. A body of mass 4 kg is projected vertically up with a velocity of 200 ms^{-1} . If it rises to a height of 800 m, the energy utilized to overcome friction is (g = 10 ms^{-2})

(A) 10 KJ (B) 48 KJ

(C) 34 KJ (D) 40 KJ

29. A raindrop of mass 2 g falling from a height of 1.00 km, hits the ground with a speed of 40.0 ms^{-1}. The work done by opposing resistive force is

(A) –18.4 J (B) –36.8 J

(C) –9.2 J (D) 18.4 J

30. A body of mass 20 kg is at rest. A force of 5 N is applied on it. The work done in first second will be

(A) 5/8 J (B) 8/5 J

(C) 5/4 J (D) 4/5 J

31. When a spring is stretched by 2 cm, its P.E. is U. If the spring is stretched by 10 cm, P.E. stored in it will be

(A) U/25 (B) U/5

(C) 5 U (D) 25 U

32. A bullet emerges out from a wooden plank with 75% of its initial kinetic energy. The total number of planks required to stop the bullet is

(A) 2 (B) 4

(C) 3 (D) 5

33. Two masses of 1g and 4g are moving with equal kinetic energies. The ratio of the magnitudes of their momenta is:

(A) $4:1$ (B) $\sqrt{2}:1$

(C) $1:2$ (D) $1:16$

34. The power of water pump is 4 kW and its efficiency is 75%. If g = 10 m/s^2, then amount of water, it can raise in 1 minute upto height of 30 m is :

(A) 100 liter (B) 200 liter

(C) 600 liter (D) 1200 liter

35. A child is sitting on a swing. Its minimum and maximum heights from the ground 0.75 m and 2 m respectively, its maximum speed will be

(A) 10 m/s (B) 5 m/s

(C) 8 m/s (D) 15 m/s

36.

A train of mass 10^4 kg is moving on a level track at a uniform speed of 72 km/hr. if the resistance due to friction is 0.25 gmwt/kg, find the power of its engine (g=9.8 m/s^2)

(A) 980W (B) 490W

(C) 245W (D) 790W

37. The work done in lifting a body of mass 20 kg and specific gravity 3.2 to a height of 8 m in water is, (g=10 ms^{-2})

(A) 110 J (B) 1600 J

(C) 500 J (D) 1100 J

38. What is the work done by a man standing with a mass of 10 Kg on his head?

(A) 1 (B) 2

(C) 3 (D) 0

39. 300 J of work is done in sliding a 2 kg block up an inclined plane of height 10 m. Work done against friction is (Take g = 10 m/s^2).

(A) 1000 J (B) 200 J

(C) 100 J (D) Zero

40. A nail is fixed at a point P vertically below the point of suspension 'O' of a simple pendulum of length 1m. The bob is released when the string of pendulum makes an angle 30^0 with horizontal. The bob reaches lowest point then describes vertical circle whose center coincides with P. The least distance of P from O is

(A) 0.4 m (B) 0.5 m

(C) 0.6 (D) 0.8 m

41. A pump fills a tank 6 × 5 × 1 m in 15 minutes. The tank is 40 m above the ground level. The power of the pump is

(A) 12 kW (B) 13 kW

(C) 0.43 kW (D) 780 kW

42.

When K.E. of a body is increases by 300 %.The momentum of the body is increases by

(A) 20 % (B) 50 %

(C) 100 % (D) 200 %

43. An open knife edge of mass M is dropped from a height 'h' on a wooden floor. If the blade penetrates a distance 's' into the wood, the average resistance offered by the wood to the blade is

(A) $Mg\left(1+\frac{h}{s}\right)$ (B) $Mg\left(1-\frac{h}{s}\right)$

(C) Mg (D) $Mg\left(1+\frac{h}{s}\right)^2$

44. Displacement of an object of mass 2 kg varies as s = 3t^2 + 5. Find work done by man from t = 0 to t = 5 sec.

(A) 100 J (B) 200 J

(C) 500 J (D) 900 J

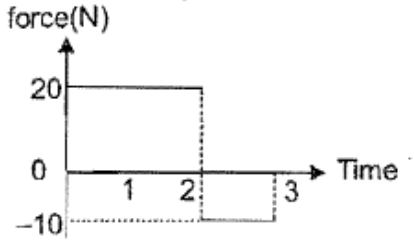
45.
Power applied to a particle varies with time as

$P = (3t^2 - 2t + 1)$ watt, where t is in second. Find the change in its kinetic energy between time t=2s and t = 4s.

(A) 32 J (B) 46 J

(C) 61 J (D) 102 J

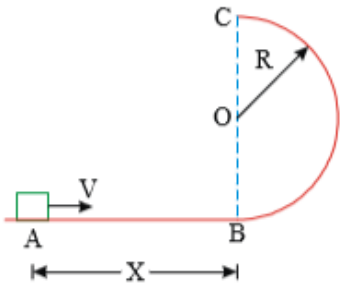
46. Starting at rest, a 5 kg object is acted upon by only one force as indicated in the figure. Find the total work done (in Joule) by the force on the object



47. A particle of mass 0.2 kg is moving in one dimension under a force that delivers constant power 0.5W to the particle. If the initial speed (in ms^{-1}) after 5s is

48. The displacement x (in m), of a particle of mass m (in kg) is related to the time t (in second) by t= √x +3. Find the work done in first six second. (in mJ)

49. A small block is given a velocity v from point A. Given x=3R, R = 20 m and g = 9.8 m/s^2. If the block strikes the point A after it leaves the smooth circular track in vertical plane, the value of v is 7x, find v?



50. Force acting on a particle is $\vec{F} = \left(\alpha y\hat{i} + \beta xy\hat{j}\right)$. Find the work done by this force, when particle is moved along the line 2x=3y from origin to the point (3, 2) {take all quantities in SI units and α = 1, β = 1 }

Chemistry XI

51. The alkene with maximum stability is

(A) $CH_3 - CH = CH - CH_3$

(B) $CH_3 - \underset{\displaystyle CH_3}{\overset{}{C}} = \underset{\displaystyle C(CH_3)_3}{\overset{}{C}} - CH_3$

(C) $(CH_3)_3C - \underset{\displaystyle C(CH_3)_3}{\overset{}{C}} \equiv \underset{\displaystyle C(CH_3)_3}{\overset{}{C}} - CH_3$

(D) $\begin{matrix} CH_3 & & CH_3 \\ & C = C & \\ CH_3 & & CH_3 \end{matrix}$

52.

$$B \xleftarrow{Lindlar/H_2} R-C \equiv C-R \xrightarrow{Na/NH_3} A$$

A, A and B are geometrical isomers (R—CH = CH —R):

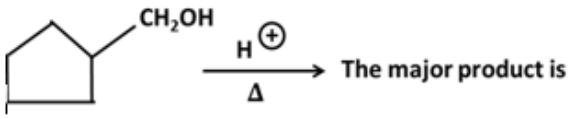
(A) A is cis, B is trans

(B) A is trans, B is cis

(C) A and B both are cis

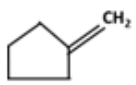
(D) A and B both are trans

53.

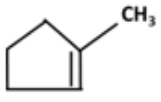


The major product is

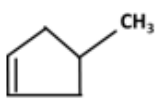
(A)



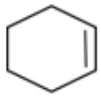
(B)



(C)



(D)



54.

$X \xrightarrow[H_2]{Pd+BaSO_4} Y, Q \xrightarrow{450^{\circ}c} Y$ where x, y & Q are gaseous hydro carbons then x, y & Q respectively are

(A) . $X - C_2H_4; Y - C_2H_6; Q - C_2H_4$

(B) $X - C_2H_2; Y - C_2H_6; Q - C_2H_4$

(C) $X - C_2H_2; Y - C_2H_4; Q - C_2H_6$

(D) $X - C_2H_6; Y - C_2H_4 - Q - C_2H_2$

55.

$CH_3 - CH_2 - CH = CH - CH_3 + O_3 \xrightarrow[Zn]{H_2O}$

(A) $CH_3 - CH_2CHO$ & CH_3CHO; each one mole

(B) Two moles of $CH_3 - CH_2CHO$

(C) Two moles of CH_3CHO

(D) $CH_3 - CH_2 - CH_2 - CH_2 - CH_3$

56. The correct order of increasing boiling points is

(A) isobutane < n-butane < n-butyl chloride < n-butanol

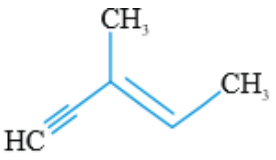
(B) isobutane < n-butyl chloride < n-butane < n-butanol

(C) n-butane < n-butanol< n-butylchloride < isobutane

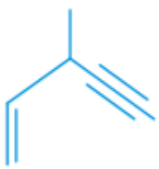
(D) n-butane< isobutane < n-butylchloride < n-butanol

57. Which would be a chiral molecule even after treatment with Lindlar catalyst?

(A)



(B)



(C)

(D)

58. The order of reactivity of the various o- and p-director groups is

(A) $-O^- > -OH > -OCOCH_3 > -COCH_3$ (B) $OH > -O^- > -OCOCH_3 > -COCH_3$

(C) $OH > -O^- > -COCH_3 > -OCOCH_3$ (D) $-O^- > -COCH_3 > -OCOCH_3 > -OH$

59.

What is the final product in the following reaction sequence?

$$R-CH_2-CH=CH_2 \xrightarrow[CCl_4]{NBS} \xrightarrow[R_2O_2]{HBr} \xrightarrow[\Delta]{Mg}$$

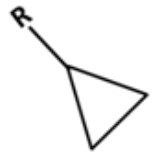
(A)

$R-CH_2=CH-CH_3$

(B)

$R-CH_2-CH-CH_3$

(C)



(D)

$$\begin{matrix} CH \\ | \\ Br \end{matrix} = C = CH\text{-}R$$

60. Octane number is zero for

(A) Isoheptane (B) n-Heptane

(C) Isooctane (D) n-Octane

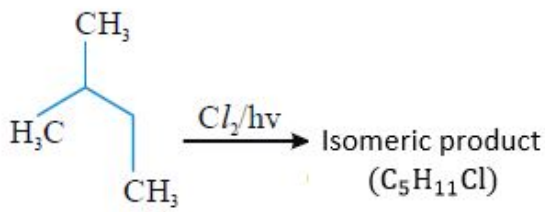
61.

t-Butyl chloride $\xrightarrow[DryEther]{Na}$ X (major)

Number of SP^2 hybridised carbons present in the product molecule.

62. The number of moles of CH_4 formed by reacting 96 grams of CH_3OH with excess CH_3MgI is

63.



(including sterioisomers)

64. The most stable oxidation state as Thallium is

(A) +1 (B) +3

(C) +4 (D) +6

65. In diborane the two H - B - H angles are nearly

(A) 60°, 120° (B) 95°, 120°

(C) 95°, 150° (D) 120°, 180°

66. The following units exist in chain silicates

(A) $(SiO_5)_n^{2n-}$ (B) $(SiO_3)_n^{3n-}$

(C) $(SiO_3)_n^{2n-}$ (D) $(SiO_6)_n^{2n-}$

67. In Borax hybridised state of boron is

(A) sp^2 (B) sp^3

(C) Both (1) and (2) (D) sp

68. Pick out the incorrect statement

(A) BO_3^{2-} ion is triangular planar in which boron is sp^2 hybridised

(B)
Boric acid contains planar triangular BO_3^{2-} units which are bonded together through
Hydrgoen bonds into two dimensional sheets

(C) Borozine is isoelectronic with benzene. In both compound π electrons are delocalised

(D) The structure of boron nitride resembles that of diamond

69. In borax the number of B – O – B links at B – OH bonds present are respectively.

(A) five at four (B) four at five

(C) three and four (D) five and five

70.

$SiF_4 + H_2O \rightarrow A \xrightarrow{1000^\circ C} B \xrightarrow{Na_2CO_3} C$ Identify B and C?

(A) H_4SiO_4 ,Na_2SiO_3 (B) SiO_2, Na_2S

(C) SiO_2, Na_2CO_3 (D) SiO_2, Na_2SiO_3

71. Ortho boric acid contains

(A) Pyramidal BO_3^{3-} units (B) Linear BO_3^{3-} units

(C) T-shaped BO_3^{3-} units (D) Triangular BO_3^{3-} units

72. Name the structure of silicate in which three oxygen atoms of $[SiO_4]^{4-}$ are shared:

(A) Pyrosilicate (B) Two dimensional sheet silicate

(C) Linear chain silicate (D) Three dimensional sheet silicate

73. The decreasing stability order of trivalent ions of group 13 is

(A) $In^{3+} > Ti^{3+} > Ga^{3+}$ (B) $Ti^{3+} > In^{3+} > Ga^{3+}$

(C) $Ga^{3+} > In^{3+} > Ti^{3+}$ (D) $Ti^{3+} > Ga^{3+} > In^{3+}$

74. What is the value of X in the given chemical formula of crystalline borax? $Na_2[B_4O_5(OH)_4]XH_2O$.

75. How many orders among following are correct with respect to the properties indicated?

(1) $NH_3 < H_2O < H_2S$, Boiling point

(2) $PH_3 < AsH_3 < NH_3 < SbH_3$ Boiling point

(3) $BF_3 < BCl_3 < BBr_3$ Extent of hydrolysis

(4) $CH_3Cl > CH_3F > CH_3Br > CH_3I$ Dipole moment

(5) $BF_3 < BCl_3 < BBr_3$ Lewis acidic strength

(6) $Na^+ > Mg^{+2} > Al^{+3}$ Extent of hydration

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KEY

1. (A)	**2.** (A)	**3.** (D)
4. (C)	**5.** (D)	**6.** (A)
7. (C)	**8.** (B)	**9.** (A)
10. (B)	**11.** [-2]	**12.** [2]
13. (A)	**14.** (B)	**15.** (C)
16. (C)	**17.** [2]	**18.** [2]
19. (B)	**20.** (A)	**21.** (B)
22. (A)	**23.** (B)	**24.** (A)
25. [8]	**26.** (C)	**27.** (C)
28. (B)	**29.** (A)	**30.** (A)
31. (D)	**32.** (B)	**33.** (C)
34. (C)	**35.** (B)	**36.** (B)
37. (D)	**38.** (D)	**39.** (C)
40. (D)	**41.** (B)	**42.** (C)
43. (A)	**44.** (D)	**45.** (B)
46. [90]	**47.** [5]	**48.** [0]
49. [5]	**50.** [7]	**51.** (D)
52. (B)	**53.** (D)	**54.** (C)
55. (A)	**56.** (A)	**57.** (B)
58. (A)	**59.** (C)	**60.** (B)
61. [2]	**62.** [3]	**63.** [6]
64. (A)	**65.** (B)	**66.** (C)

67. (C) **68.** (D) **69.** (A)

70. (D) **71.** (D) **72.** (B)

73. (C) **74.** [8] **75.** [4]

SOLUTIONS

1.

$$\lim_{x\to 0}\frac{(1-\cos x)-\cos 2x(1-\cos x)}{x^4}=\lim_{x\to 0}\frac{(1-\cos x)(1-\cos 2x)}{x^4}$$

$$\lim_{x\to 0}\frac{2\sin^2\left(\frac{x}{2}\right)}{x^2}\times\frac{2\sin^2 x}{x^2}$$

$$\lim_{x\to 0}(4).\left(\frac{\sin\left(\frac{x}{2}\right)}{\left(\frac{x}{2}\right)}\right)^2\times\left(\frac{\sin x}{x}\right)^2\times\left(\frac{1}{2}\right)^2$$

$$4.\ (1)^2\times(1)^2\times\frac{1}{4}=1$$

2.

$$\left(\frac{0}{0}\right),\quad by\ \ LH\ rule$$

$$\lim_{x\to a}\frac{f(a)g^1(x)-g(a)f^1(x)}{g^1(x)-f^1(x)}=4\quad(\therefore\ f(a)=f=g(a)=K$$

$$\Rightarrow\frac{f(a)g^1(a)-g(a)f^1(a)}{g^1(a)-f^1(a)}=4$$

$$\Rightarrow k\left[\frac{g^1(a)-f^1(a)}{g^1(a)-f^1(a)}\right]=4$$

$$\Rightarrow\ K=4$$

3.

$$f(x)=\cos x(x^2-2x^2)-x(2\sin x-2x\tan x)+1(2\sin x.x-x^2\tan x)$$

$$f(x)=-x^2\cos x-2x\sin x+2x^2\tan x+2x\sin x-x^2\tan x$$

$$=f(x)=-x^2\cos x+x^2\tan x;$$

$$f^1(x)=x^2\sin x+\cos x(-2x)+x^2\sec^2 x+\tan x(2x)$$

$$\lim_{x\to 0}\frac{f^1(x)}{x}=\lim_{x\to 0}(x\sin x-2+x\sec x+2\tan x)$$

$$=0-2(1)+0+0=-2$$

4.

$$\lim_{x\to\infty}\frac{1+\frac{1}{3}+\frac{1}{9}+\cdots+\frac{1}{3^n}}{1+\frac{1}{5}+\frac{1}{5^2+\cdots+\frac{1}{5^n}}}\qquad(s_n\ in\ G.P)$$

$$\lim_{x\to\infty}\frac{(1)\left(\frac{1-\frac{1}{3}n+1}{1-\frac{1}{3}}\right)}{(1)\left(\frac{1-\frac{1}{3}n+1}{1-\frac{1}{5}}\right)}=\frac{\frac{1}{2}}{\frac{1}{4}}=\frac{\frac{3}{2}}{\frac{5}{4}}=\frac{3}{2}\times\frac{4}{5}=\frac{6}{5}$$

8.

$$\frac{dx}{dt} = -\sin t, \quad \frac{dy}{dt} = \frac{1}{t}$$

$$\frac{dy}{dx} = \frac{-1}{t\sin t}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dt}\left(\frac{dy}{dx}\right).\frac{dt}{dx}$$

$$= \left(\frac{\sin t + t\cos t}{t^2\sin^2 t}\right).(-\operatorname{cosec} t)$$

$$\Rightarrow \qquad \text{at } t = \frac{\pi}{2}, \quad \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -\frac{4}{\pi^2} + \frac{4}{\pi^2} = 0$$

9.

$y \ln x + x \ln y = x^2$

$\Rightarrow \quad \frac{y}{x} + y' \ln x + \frac{x}{y}y' + \ln y = 2x$

$\Rightarrow \quad y' = \frac{2x^2y - y^2 - xy\ln y}{xy\ln x + x^2}$

$y'(1) = e - e^2$

$(\because y = e \text{ at } x = 1)$

10.

$$\lim_{x\to\infty} \frac{1-\cos(ax^2+bx+1)}{(x-\alpha)^2} = \lim_{x\to\infty} \frac{1-\cos(a(x-\alpha)(x-\beta))}{(x-\alpha)^2}$$

$$= \lim_{x\to\infty} \frac{1-\cos(a(x-\alpha)(x-\beta))}{[(x-\alpha)(x-\beta)]^2} \times (x-\beta)^2$$

$$= \frac{a^2}{2}(\alpha-\beta)^2$$

11.

Since f(x) is odd, f(–x) = – f(x) or f′(–x) (–1) = –f′(x)

Or f′(–x) = f′(x)

∴ f′(–3) = f′(3) = –2

12.

$$y = \sin^{-1}\left[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\right] \text{ where } 0<x<1$$

$$= \sin^{-1}\left[x\sqrt{1-\left(\sqrt{x}\right)^2} - \sqrt{x}\sqrt{1-x^2}\right]$$

$$= \sin^{-1}x - \sin^{-1}\sqrt{x}$$

[Using $\sin^{-1}x - \sin^{-1}y = \sin^{-1}(x\sqrt{1-y^2} - y\sqrt{1-x^2})$]

Differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-\left(\sqrt{x}\right)^2}}\frac{d}{dx}\left(\sqrt{x}\right)$$

$$= \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}}.$$

13.

f(1) = a + b

$f(1+h) = \frac{|1+h-1|}{1-(1-h)} + a = -1 + a$

∵ Function is continuous

∴ f(1) = f(1 + h)

= a + b = −1 + a ⇒ b = −1

Now $f(1-h) = \frac{|1-h-1|}{1-(1-h)} + b = \frac{h}{h} + b = 1 + b$

∴ a + b = 1 + b ⇒ a = 1

14.

LHL (X = 0) = f (0) = RHL (x = 0)

$LHL = \lim_{x\to 0^-} \frac{\sqrt{1+px}-\sqrt{1-px}}{x} = \frac{2p}{2} = p$

$f(0) = -\frac{1}{2} = \text{RHL}$

15. $h = \sqrt{2}R$

16. $f(x)$ is continuous at $x = 0$

17.

$$\frac{dy}{dx}=\frac{9t^2-1}{6t}$$

$$x=-2, y=2 \rightarrow t=-1, \frac{dy}{dx}=\frac{-4}{3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$\frac{dy}{dx}=\frac{9\left(\frac{4}{9}\right)-1}{6\left(\frac{2}{3}\right)}=\frac{3}{4} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

From (1) and (2) tangents at P and Q are perpendicular

18. at x = 1, x = 2, fof is discontinuous

19.

Slope of normal $m = tan\theta$

Equation of normal at $(a + acos\theta, a\, sin\theta)$ is

$$y - asin\theta = \frac{sin\theta}{cos\theta}(x - a - acos\theta)$$

$$xsin\theta - ycos\theta = a \sin\theta$$

It is always through $(a, 0)$

20.

$$f(x) = 2x^3 - 21x^2 + 36x + 20$$

$$f'(x) == 6x^2 - 42x + 36 \Rightarrow f'(x) = 0$$

f has maxima or minima

21.

$$P = 2r = \theta r = r(2 + \theta) \Rightarrow r = \frac{P}{2+\theta}$$

22.

Find slope m and apply formula $\frac{y^2(1+m^2)}{2|m|}$

23. $$f'(c) = \frac{log\left(sin\frac{5\pi}{6}\right) - log\left(sin\frac{\pi}{6}\right)}{\frac{5\pi}{6} - \frac{\pi}{6}}$$

24.

Since $f(1-0) = \lim_{x\to 1} 3^x = 3$

$f(1,0) = \lim_{x\to 1}(4-x) = 3$

and $f(1) = 3^1 = 3$

$\therefore f(1-0) = f(1+0) = f(1)$

$\Rightarrow$ f(x) is continuous at x = 1

Again $f'(1+0) = \lim_{x\to 1^+} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1} \frac{3^x-3}{x-1}$

$= \lim_{h\to 0} \frac{3^{1+h}-3}{h} = 3\lim_{h\to 0} \frac{3^h-1}{h} = 3\log 3$

and $f'(1+0) = \lim_{x\to 1} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1} \frac{4-x-3}{x-1} = -1$

$\therefore f'(1+0) \neq f'(1-0)$

$\Rightarrow f(x)$ is not differentiable at x = 1.

25.

(i) $f(x) = \sin^{-1}(2x\sqrt{1-x^2}) = \begin{cases} -\pi - 2\sin^{-1}x, & -1 \le x < -\frac{1}{\sqrt{2}} \\ 2\sin^{-1}x, & -\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}} \\ \pi - 2\sin^{-1}x, & \frac{1}{\sqrt{2}} < x \le 1 \end{cases}$

$f'(x) = \begin{cases} -\frac{2}{\sqrt{1-x^2}}, & -1 < x < -\frac{1}{\sqrt{2}} \\ \frac{2}{\sqrt{1-x^2}}, & -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}} \\ \frac{-2}{\sqrt{1-x^2}}, & \frac{1}{\sqrt{2}} < x < 1 \end{cases}$

$f'(1/2) = \frac{4}{\sqrt{3}}, \quad f'(-1/2) = \frac{4}{\sqrt{3}}$

26.

Kinetic energy $\frac{1}{2}mv^2$As both balls are falling through the same height, therefore the possess same velocity but KE $\propto$ m (If v = constant)

$\therefore \frac{(KE)_1}{(KE)_2} = \frac{m_1}{m_2} = \frac{2}{4} = \frac{1}{2}$

27. $p'_{required} = \frac{d(mgh)}{dt}$

$= \frac{9000 \times 10 \times 120}{60 \times 60}$

$= 3000\ w$

$It\ accounts\ of\ \frac{3}{4}\ p = p'$

$p = \frac{4}{3} \times 3000$

$= 4000\ W$

28. $\Delta E = \frac{1}{2}mv^2 - mgh$

29. $W = -\left|mgh - \frac{1}{2}mv^2\right|$

30.

$$a = \frac{F}{m} = \frac{1}{4}mv^2$$
$$v = at = \frac{1}{4} \times 1$$
$$= \frac{1}{4} m/s$$
$$w = \frac{1}{2}mv^2 = \frac{1}{2}(20)\left(\frac{1}{16}\right)$$
$$= \frac{5}{8} J$$

31. $v \propto x^2$

$v' = 25\,U$

32. $\frac{25}{100} = \frac{1}{x}$

$x = 4$

33. $\frac{P_1^2}{2m_1} = \frac{P_2^2}{2m_2}$

$$\left(\frac{P_1}{P_2}\right)^2 = \frac{1}{4}$$

$$\frac{P_1}{P_2} = \frac{1}{2}$$

34.

Efficiency = 75%

$$75 = \frac{\text{Output power}}{\text{Input power}} \times 100$$

$$= \frac{mg/t}{4000} \times 100$$

$$= \frac{m \times 10 \times 30}{4000 \times 260} \times 100$$

$$= 75 = \frac{m}{8} \Rightarrow m = 600\text{kg} = 600 \text{ liter}$$

35. Drop in potential energy = maximum kinetic energy

37.

$$w = mgh\left[1 - \frac{1}{n}\right]$$

$$= 20 \times 10 \times 8\left[1 - \frac{1}{3.2}\right]$$

$$= 1600\left[\frac{2.2}{3.2}\right]$$

$$= \frac{11}{16} \times 1600$$

$$= 1100 \text{ J}$$

38. Conceptual

40.

If V is velocity at the lowest point

$V^2 = 2gL(1-\sin q)$

but, $V^2 = 5gr$ and r = L - x

where x is minimum distance between 0 and P.

find x.

41. $p = \frac{mgh}{t}$

$= \frac{30 \times 10 \times 40}{15} = \frac{400}{30}$

$= \frac{40}{3}$

42. $k = \left(\frac{1}{2}mv^2\right) \times \frac{m}{m}$

$k = \frac{p^2}{2m}$

$10g\, k = 2\log p + \log 2m$

$Differentiate$

$\frac{1}{k} = 2\frac{dp}{p} + 0$

$\frac{200k}{k} = 2\frac{dp}{p}$

$dp = 100p$

43. $Mg(h+s) = Fs$

$$F = mg\left[1 + \frac{h}{s}\right]$$

44.

V = $\frac{ds}{dt} = 6t$

Initial kinetic energy = $1/2 \times 2 \times (0)^2$

Final kinetic energy = $1/2 \times 2 \times (30)^2$

W = $k_t = k_t = 900$ J

45. $p = \frac{dw}{dt} = 3t^2 - 2t + 1$

$$w = \int dw = \int_2^4 (3t^2 - 2t + 1)\, dt$$

$$= \left(t^3 - t^2 + t\right)_2^4$$

$$= (64 - 16 + 4) - (8 - 4 + 2)$$

$$= 46\ j$$

46. Change in velocity = $\frac{area\ under F - T graph}{mass} = \frac{40 + (-10)}{5}$ = 6m/s $\Rightarrow$

W_F = DK.E. $= \frac{1}{2}(5)6^2$ = 90J

47.

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = W$$

$$\frac{1}{2}mv^2 = Pt$$

$$v = \sqrt{\frac{2Pt}{m}} = \sqrt{\frac{2\times0.5\times5}{0.2}} = 5ms^{-1}$$

48.

$$x = (t-3)^2 = t^2 - 6t + 9$$

$$v = \frac{dx}{dt} = 2t - 6$$

at $t = 0, v = -6$; at $t = 6, v = +6$

initial $KE = \frac{1}{2}m(-6)^2 = 18m$

final $KE = \frac{1}{2}m(6)^2 = 18m$

49. Conceptual

50.

$$W = \int \vec{F} . \overrightarrow{dr}$$

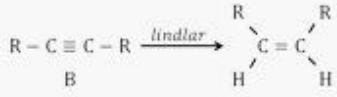
$$= \int (ydx + xydy) \qquad \because\ 2x = 3y$$

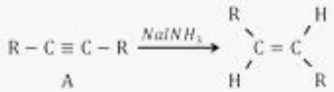
$$= \int_0^2 \left(\frac{3}{2}ydy + \frac{3}{2}y^2dy\right) \qquad \therefore\ dx = \frac{3}{2}dy = \left[\frac{3}{4}y^2 + \frac{y^3}{2}\right]_0^2 = 7\ Joule$$

$$B \xleftarrow{lindlar} R-C\equiv C-R \xrightarrow{NaINH_3} A$$

A & B are geometrical isomers;

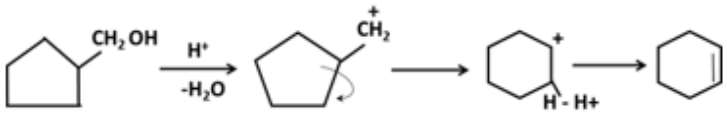
A is trans





52. B is Cis

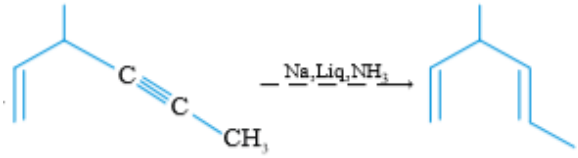
53.



54. Conceptual

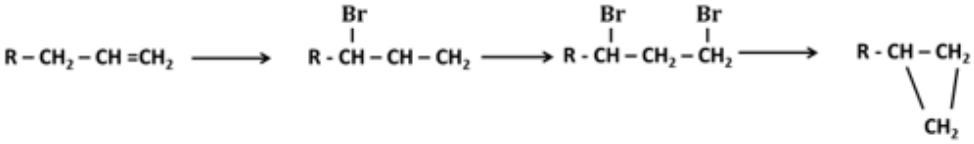
55. Conceptual

57.



58. $-O^- > _{OH} > -OCOCH_3 > -COCH_3$

59.



61. Conceptual

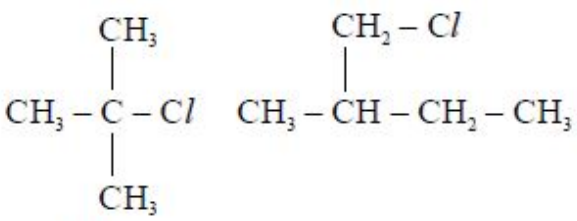
62.

$$CH_3OH + CH_3MgI \rightarrow CH_4 + Mg(OH)I$$

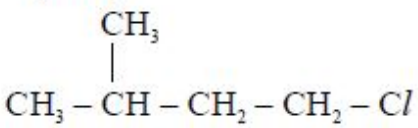
32 g 1 *mole*

96 g 3 *mole*

63.



(d & l)



$CH_3CH(CH_3)CH(Cl)CH_3$ and its mirror image

Total isomeric products = 6

64. Due to the inert pair effect Tl has stable oxidation state Tl^{+1} like Alkali metals.

65. H-B-H angles are 95°, 120° on diborane.

72. Two dimensional sheet structures are formed when three oxygen atoms of each $[SiO_4]^{4-}$ tetrahedral are shared.

74. $Na_2[B_4O_5(OH)_4].8H_2O$