GMR Classes

JEE MAIN - MOT - 33 - DT 04-04-2020 - Keys And Solutions

Total Marks : 300
Duration : 3:00 hrs

KEY

1. (B)	**2.** (A)	**3.** (A)
4. (C)	**5.** (A)	**6.** (D)
7. (D)	**8.** (C)	**9.** (A)
10. (B)	**11.** (B)	**12.** (C)
13. (C)	**14.** (A)	**15.** (B)
16. (C)	**17.** (A)	**18.** (D)
19. (A)	**20.** (A)	**21.** [0]
22. [4]	**23.** [6]	**24.** [6]
25. [3]	**26.** (A)	**27.** (C)
28. (C)	**29.** (A)	**30.** (B)
31. (A)	**32.** (C)	**33.** (A)
34. (D)	**35.** (D)	**36.** (A)
37. (B)	**38.** (A)	**39.** (B)
40. (C)	**41.** (A)	**42.** (D)
43. (C)	**44.** (A)	**45.** (C)
46. [1]	**47.** [2]	**48.** [0]
49. [1]	**50.** [3]	**51.** (A)
52. (A)	**53.** (C)	**54.** (D)
55. (B)	**56.** (A)	**57.** (C)
58. (A)	**59.** (C)	**60.** (C)
61. (B)	**62.** (A)	**63.** (C)

64. (B)	**65.** (D)	**66.** (B)
67. (D)	**68.** (A)	**69.** (D)
70. (D)	**71.** [8]	**72.** [10]
73. [5]	**74.** [3]	**75.** [3]

SOLUTIONS

1. $\left(1 + z + z^2\right)^8 = c_0 + c_1z + \dots + c_{16}z^{16}$

take $z = i$

$\Rightarrow (1 + i - 1)^8 = c_0 + c_1i - c_2 - c_3i + \dots + c_{16}$

$1 = c_0 + c_1i - c_2 - c_3i + c_4 + \dots + c_{16}$......(1)

Take, $z = -i$

$\Rightarrow (1 - i - 1)^8 = c_0 - c_1i - c_2 + c_3i + c_4 + \dots + c_{16}$

$1 = c_0 - c_1i - c_2 + c_3i + c_4 + \dots + c_{16}$.... (2)

Add (1) & (2)

$2 = 2[c_0 - c_2 + c_4 - c_6 + \dots + c_{16}]$

$\Rightarrow c_0 - c_2 + c_4 - c_6 + \dots + c_{16} = 1$

3. $x^2 - 2x + 4 = 0$

$$x = \frac{2 \pm \sqrt{4 - 16}}{2} = \frac{2 \pm 2i\sqrt{3}}{2} = 1 \pm i\sqrt{3}$$

$\alpha = 1 + i\sqrt{3}$, $\beta = 1 - i\sqrt{3}$

4. By Verification

5. Given that $z = \sum_{r=1}^{\infty} \sin\left(\frac{2r\pi}{9}\right) + i\cos\left(\frac{2r\pi}{9}\right)$

$$\Rightarrow z = \sum_{r=1}^{\infty} i.\, e^{\frac{-i2r\pi}{9}}$$

let $e^{\frac{-i2\pi}{9}} = w$

$$\Rightarrow z = \sum_{r=1}^{\infty} i.\, w^{r}$$

$$\Rightarrow z = i[w + w^2 + w^3 + \ldots. + w^8]$$

$$\Rightarrow z = iw\left[\frac{1 - w^8}{1 - w}\right] = \frac{iw - iw^9}{1 - w}$$

Since $w^9 = \cos 2\pi - i\sin 2\pi = 1$

$$\Rightarrow z = \frac{iw - i.1}{1 - w} = \frac{i(w - 1)}{1 - w} = -i$$

Therefore $z = -i$

Now, $z + \bar{z} = -i + i = 0$

and $z\bar{z} = -i \times i = -i^2 = 1$

$\therefore\ z + \bar{z} = 0$ and $z\bar{z} = 1$

6. since the equation has real solution

$Z = x + iy = x$

$x^2 + (p + iq)\,x + irs = 0$

$x^2 + px + r = 0$ and $qx + s = 0$

From 2nd, $x = -s/q$ and putting in first, we get

$pqs == s^2 + q^2r$ which is the required condition

7.

$$17 \equiv 2(\bmod 5)$$
$$17^5 \equiv 2^5(\bmod 5) \equiv 2(\bmod 5)$$
$$\Rightarrow \left(17^5\right)^6 = 2^6(\bmod 5) \Rightarrow (17)^{30} \equiv 4(\bmod 5)$$

8. $\left(\frac{1+i}{1-i}\right)^4 + \left(\frac{1-i}{1+i}\right)^4 = i^4 + (-1)^4 = 1 + 1 = 2$

9. $\frac{n}{2} < a(n) < n$

10. Put n = 1 and verify the options

11. $\frac{\pi^c}{2}$

12.

let $z = x + iy, z_1 = x + iy$ and $z_2 = x_2 + iy_2$

G T $z + \overline{z} = 2|z - 1|$ and $\arg(z_1 - z_2) = \frac{\pi}{4}$

$\Rightarrow 2x = 1 + y^2 \ldots 1$

$\frac{y_1 - y_2}{x_1 - x_2} = 1$

since z_1 & z_2 both satisfy 1 we have

$2x_1 = 1 + y_1^2 .. \& 2x_2 = 1 + y_2^2$

$2(x_1 - x_2) = (y_1 + y_2)(y_1 - y_2)$

$2 = (y_1 + y_2)\left(\frac{y_1 - y_2}{x_1 - x_2}\right)$

$\Rightarrow y + y_2 = 2$

13. z_1, z_2, 0 will be the vertices of an equilateral triangle

if $z_1^2 + z_2^2 + 0^2 = z_1 z_2 + 0 z_2 + 0 z_1$

$\Rightarrow 1 - \frac{2b}{3} = \frac{b}{3} \Rightarrow b = 1$

14.

$1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \ldots\ldots + (2n+1)^2$

$= 1^2 + 2^2 + 3^2 + \ldots\ldots + (2n+1)^2$

$-2\left[2^2 + 4^2 + \ldots\ldots + (2n)^2\right]$

$= \frac{(2n+1)(2n+2)(4n+3)}{6}$

$- \frac{8.n(n+1)(2n+1)}{6} = (n+1)(2n+1)$

15. $T_n = (n + 1)(2^n - 1)$

16. P(n) = $n^2 - n + 41$

P(5) = 25 – 5 + 41 = 61

P(3) = 9 – 3 + 41 = 47

Both true

17. $\left(\frac{1 + \cos\theta + i\sin\theta}{i + \sin\theta + i\cos\theta}\right)^4 = \left[\frac{1 + e^{i\theta}}{i + ie^{-i\theta}}\right]^4$

$= \frac{1}{i^4} \times \left[\frac{1 + e^{i\theta}}{1 + e^{-i\theta}}\right]$

$= \left[\frac{1 + e^{i\theta}}{e^{i\theta} + 1} \times e^{i\theta}\right]$

$= e^{i\theta}$

$= \cos\theta + i\sin\theta$

$\therefore \cos\theta + i\sin\theta = \cos n\theta + i\sin n\theta$

$\Rightarrow n = 1$

18. $2^3\left(1^3 + 2^3 + 3^3 + \ldots + n^3\right) = kn^2(n + 1)^2$

19. First term of 50th bracket = (1+2+3+ .... +49) +1 = 1226

20. The values of Z for which |Z + i| = |Z − i| are

Let Z = a + ib

$\Rightarrow$ |a+ib + i| = |a+ib − i|

$\Rightarrow$ |a+i(b+1)| = |a+i(b−1)|

$\Rightarrow a^2 + (b+1)^2 = a^2 + (b-1)^2$

$\Rightarrow (b+1)^2 = (b-1)^2$

$\Rightarrow b^2 + 1 + 2b = b^2 - 2b + 1$

$\Rightarrow$ 4b = 0

$\Rightarrow$ b = 0

Therefore, This is true for any real number.

21.

$$|z_1 + z_2 + \dots\dots + z_n|^2 = 0$$

$$\Rightarrow |z_1|^2 + |z_2|^2 + \dots\dots + |z_n|^2 + z_1\bar{z}_2 + z_2\bar{z}_1 + \dots\dots = 0 \qquad \to (1)$$

Also $P = \sum_{j=1}^{n}\sum_{i=1}^{n}\left(\frac{z_i}{z_j}\right)$

$$= n + \frac{z_1\bar{z}_2}{|z_2|^2} + \frac{z_2\bar{z}_1}{|z_1|^2} + \dots$$

$P = n + \frac{(z_1\bar{z}_2 + z_2\bar{z}_1 + \dots\dots)}{r^2}$, where $|z_1| = |z_2| = \dots\dots = |z_n|$

$$\Rightarrow \mathrm{Re}(P) = n + \frac{\mathrm{Re}(z_1\bar{z}_2 + z_2\bar{z}_1 + \dots\dots)}{r^2}$$

$= n + \frac{(-nr^2)}{r^2} = 0$. (Using (1))

Or

$z_1, z_2, z_3 \dots\dots z_n$ can be taken as n^{th} roots of unity

22.

Let $z = x + iy$

$\Rightarrow \quad x^2 - y^2 + 2ix\,y = x - iy$

$\Rightarrow \quad x^2 - y^2 = x \quad$ and $\quad 2xy = -y$

From (2), $2xy = -y \quad \Rightarrow \quad y = 0,\ x = \frac{-1}{2}$

When $y = 0$ from (1), we get $x = 0, 1$

when $x = \frac{-1}{2} \quad \Rightarrow \quad y = \pm\frac{\sqrt{3}}{2}$

Hence the solution of the equation are

$z_1 = 0 + 1.0$, $z_2 = 1 + 1.0 = 1$

$z_3 = \frac{-1}{2} + \frac{\sqrt{3}}{2}i$ and $z_4 = \frac{-1}{2} - \frac{\sqrt{3}}{2}i$

Hence number of solution is 4

23.

Hint: Let $A(1,2)$ and $B(-1,4)$ be the given points.

The equation of the perpendicular bisector of $\overline{AB}$ is $x - y + 3 = 0$.

Given line is $z(1+i) + \bar{z}(1-i) + K = 0$

$\Rightarrow z(1+i) + \overline{z(1+i)} + K = 0$

$\Rightarrow 2\text{Re}(z(1+i)) + K = 0$

$\Rightarrow 2\text{Re}\left[(x+iy)(1+i)\right] + K = 0$

$\Rightarrow 2(x-y) + K = 0$

$\Rightarrow x - y + \frac{K}{2} = 0$

$\therefore \frac{K}{2} = 3 \Rightarrow K = 6$.

24.

$z_3 - z_1 = 2\left(\frac{1-i\sqrt{3}}{2}\right)(z_2 - z_1)$

or, $z_3 - z_1 = 2(-\omega)(z_2 - z_1)$

$= 2(z_2 - z_1)\, e^{-i\pi/3}$

Let $|z_2 - z_3| = \alpha$

Using cosine rule in ΔABC $\frac{1}{2} = \frac{x^2 + 4x^2 - \alpha^2}{2.x.2x}$

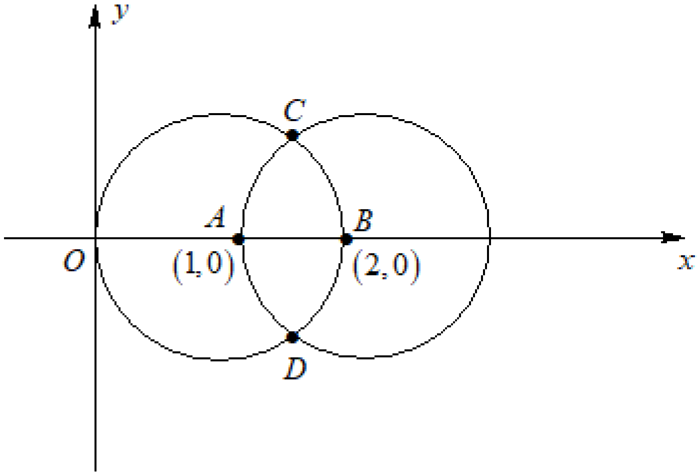
or, $3x^2 = \alpha^2$

Area $= \Delta = \frac{1}{2}\,.\,2x\,.\,x.\,\sin \pi/3$

$= \frac{\sqrt{3}}{2}.\frac{\alpha^2}{3}$ or, $\sqrt{3}\,|z_2 - z_3|^2 = 6\Delta \therefore k = 6.$

25.

Hint: $|z-1|\le 1$ represents the interior and boundary of the circle with centre at $1+0i$ and radius 1 and $|z-2|=1$ represents the circle with centre at $2+0i$ and radius 1.



Clearly the points z satisfying $|z-1|\le 1$ and $|z-2|=1$ lie on the arc DAC.

$\therefore \quad OA \le |z| \le OC(=OD)$

As $\angle OCB = \pi/2, OC^2 = OB^2 - BC^2 = 4-1=3$

$\Rightarrow \quad OC=\sqrt{3}$

Thus, $|z|^2 \le 3$.

26.

$$S = \tfrac{1}{2}a_{net}t^2 + (0)t \qquad (\because u=0)$$

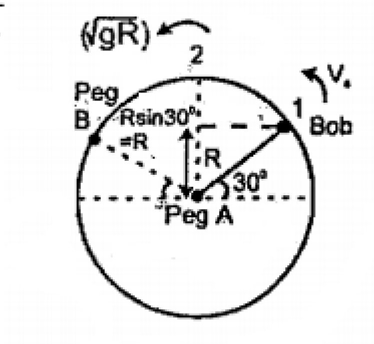
$$\therefore \frac{l}{2} = \frac{1}{2}(a-\mu g)t^2$$

$$\therefore t = \sqrt{\frac{l}{a-\mu g}}$$

27.

For anti-clockwise motion, speed at the highest point should be $\sqrt{gR}$

Conserving energy at (1) & (2) :



$$\frac{1}{2}mv_a^2 = mg\frac{R}{2} + \frac{1}{2}m(gR)$$

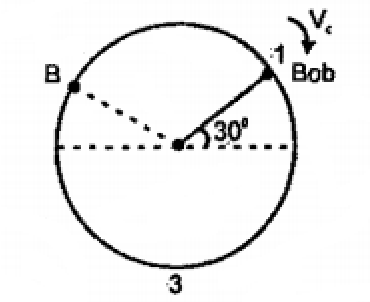
$\Rightarrow \quad v_a^2 = gR + gR = 2gR \quad \Rightarrow \quad v_a = \sqrt{2gR}$

For clock-wise motion, the bob must have atleast that much speed initially, so that the string must not become loose any where until it reaches the peg B. At the initial position :

$T + mg\cos60^\circ = \frac{mv_c^2}{R}$;

v_c being the initial speed in clockwise direction.

For $\quad v_{C\,min}$: Put T = 0 ;



$\Rightarrow \quad v_C = \sqrt{\frac{gR}{2}} \quad \Rightarrow \quad v_C/v_a = \frac{\sqrt{\frac{gR}{2}}}{\sqrt{2gR}} = \frac{1}{2}$

$\Rightarrow \quad v_C : v_a = 1 : 2 \quad$ **Ans.**

28. $A = \frac{1}{2}r^2\theta$

$$\frac{dA}{dt} = \frac{1}{2}r^2\frac{d\theta}{dt}$$

$$\frac{1}{2}r^2\omega$$

30.

From the relation;

$F - \mu mg = ma$

$a = \frac{F - \mu mg}{m} = \frac{129.4 - 0.3 \times 9.8}{10}$

$= 10\ m/s^2$

31.

With respect to platform the initial velocity of

the body of mass m is 4 m/s towards left and it starts retarding at the rate of $a = 2m/s^2$

Using $v^2 = u^2 + 2as$ we get:

$0^2 = 4^2 + 2(-2)(s) \Rightarrow s = 4^2 + 2(-2)(s) \Rightarrow s = 4$ meter.

32. $F = mg + \mu N$

$F = 2 + (0.4)(5)$,

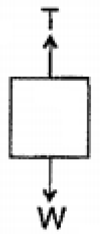
$\therefore F = 4N$

33.

As tengential acceleration a = dv/dt = ωdr/dt

but ω = 4π and dr/dt = 1.5 (reel is turned uniformly at the rate of 2 r.p.s.)

∴ a = 6π , Now by the F.B.D. of the mass.



$$T - W = \frac{W}{g}a$$

∴ T = W (1 + a/g) put a = 6π

∴ T = 1.019 W

$\mu mg = m\,\omega_1^2 r_1$

$\omega_{1}^{2}{}_{1} = (2\omega_1)^2\, r_2$

$\frac{20}{4} = r_2$

34. $r_2 = 5\ cms$

36.

Let m is the mass per unit length, then rate of mass

Per sec $= \frac{mx}{t} = mv$

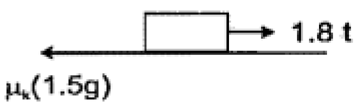
Rate of KE=$\frac{1}{2}(mv)v^2 = \frac{1}{2}mv^3$

37.

$1.8\,t - \mu_k\,15 = 1.5\,(1.2\,t - 2.4)$

For t = 2.85 sec.

$\mu_k = 0.24$



Tangential acceleration $= g\sin\theta$

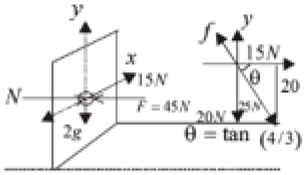
Radial acceleration $= \frac{v^2}{l}$

$V = \sqrt{2gh} = \sqrt{2g(l - l\cos\theta)}$

$g\sin\theta = 2g\,(1 - \cos\theta)$

38. on solving $\cos\theta = \frac{3}{5}$ $\theta = 53°$

39.



$f_{\max} == \mu N = (0.5)(45) = 22.5$ newton

Since magnitude of net external force expect friction is 25 N, therefore,

$= 25$ N

$$|a| = \frac{25 - 22.5}{2} = 1.25\ m/s^2$$

41.

Given Data:

$F_1 = 50\ N$

$a_1 = 2\ m/s^2$

$F_2 = 200\ N$

$a_2 = 3\ m/s^2$

$m = ?$ and $\mu_k = ?$

$$\frac{F - f}{m} = a$$

$$\frac{150 - f}{m} = 2 \Rightarrow 150 - f = 2m \rightarrow (1)$$

$$\frac{200 - f}{m} = 3 \Rightarrow 200 - f = 3m \rightarrow (2)$$

By solving (1) & (2) we get $m = 50\ N$

We know that, $F = \mu m g$

$50 = \mu(50)(10)$

$\therefore \mu = 0.1$

Hence option (A) is correct.

42. *initial energy = final energy*

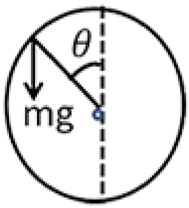
$mgh = \frac{1}{2}m\left(\frac{7}{2}gl\right)$

$h = \frac{7}{4}l$

$= l + \frac{3}{4}l$

Final

$T + mg\cos\theta = \frac{mv^2}{r}$



As string shakens $T = 0$

$mg\cos\theta = \frac{m}{1}(u^2 - 2gl)$

$mg\cos\theta = m\left[\frac{7}{2}gl - 2g(1 + \cos\theta)\right]$

$\cos\theta = \frac{1}{2}s$

$\theta = 60^0$

$\theta' = 180 - 60^0 = 120^0$ *with vertical*

$v \propto \frac{1}{h^2}$

$r \propto \frac{1}{n^2}$

43. $\therefore$ centripetal force = 16F

$mg = 4 \times 9.81\ N$

$T = \ 103.2N$

$\frac{mv^2}{r} = \frac{4\times4\times4}{1} = 64$

We notice

$\frac{mv^2}{r} + mg = T$

$i.e. T - mg = \frac{mv^2}{r}$

44. $i.e. \theta = 0$

$a_t = r_\alpha$

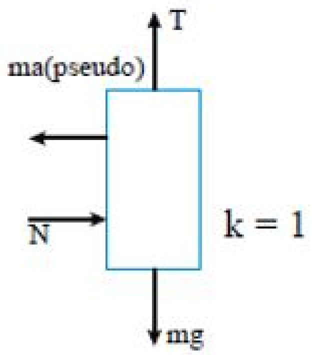
$a_c = \omega^2 r$

α, ω are same for both

45. $\therefore$ P has more acculation as $r_p > r_q$

46.

Block C, N = ma Frictional force = μma



47.

If $mg > \frac{mv^2}{r}$, friction force acts outwards

$T - \mu mg = \frac{mv^2}{r_{\max}}$

If $mg < \frac{mv^2}{r}$ fractional force on car towards centre

$T + \mu mg = \frac{mv^2}{r_{\min}}$

48.

Block will start when $4t = \mu_s mg$

So, $t = \mu_s \frac{mg}{4} \Rightarrow t = \frac{0.6 \times 6 \times 10}{4} = 9s$

So, at t = 5 sec block will be stationary

49.

The mass m is acted by two forces

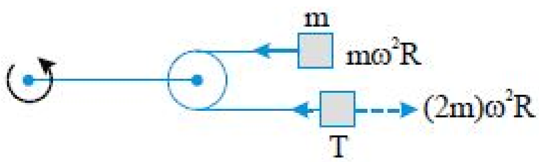
i) weight mg a mass M ii) Tension T isthe string

$T\cos\theta = mg$; $T\sin\theta = mr\omega^2$ and $T = Mg$

$r = l\sin\theta$; $Mg\sin\theta = m(l\sin\theta)\omega^2$

$\omega^2 = \frac{Mg}{ml}$; $n = \frac{1}{2\pi}\sqrt{\frac{Mg}{ml}}$

50.



By Newton's second law, we can write

$(2m)\omega^2 R - T = (2m)a$ ------- (i)

and $T - m\omega^2 R = ma$ ------- (ii)

After simplifying, above equation, we get

$a = 300 m/s^2$

$\therefore a = 3 \times 100 \Rightarrow n = 3$

52. Conceptual

54. $C = 1/\sqrt{d}$

55.

$P_1V_1 = P_2V_2$ let initial pressure = 100 units, then

$100 \times V_1 = 101 \times V_2 \Rightarrow V_2 = \frac{100}{101}V_1$

Decease in volume $= V_1 = \frac{100}{101}V_1 = \frac{1}{101}V_1$

% decrease $= \frac{100}{101}\%$

56. $\Delta H - \Delta U = \Delta nRT$

57.

Because of more molecular weight of N_2, should have more temperature in order to get some r.m.s velocity as Hydrogen. So

$T_{H_2} < T_{N_2}$

60.

$$T_C = \frac{89}{27Rb}$$

$$T_B = \frac{a}{R_b} \quad \text{substitute } T_B \text{ in } T_C$$

$$\text{Then} \Rightarrow T_C = \frac{8}{27Rb}(T_B Rb)$$

$$T_C = \frac{8}{27}T_B$$

61.

Density of a gas is given $\rho = \frac{PM}{RT}$. Obviously the choice that has greater $\frac{P}{T}$

62.

Diagram

$\Delta H_{Rxn} = \Delta^{\circ} H_{f(cyclohexone)} - \Delta^{\circ} H_{f(benzene)}$

$= -156 - 49 = 205$ KJ

According reduction of cyclohexane it requires 3×119 KJ of energy is need to reduce the benzene to cyclohexane

Then resonance energy of benzene $= -3 \times 119$ (Theritial) + 205(practical)

$= -152$ KJ/ mole

63. The heat of neutralization for any strong acid and strong base is -13.7 k Cal/mole at 25^0 C

64.

$$\left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT$$

attractive forces represented by $\frac{an^2}{V^2}$

65.

Rate of diffusion $\propto \frac{1}{\sqrt{M.W}}$

So, the gases which have same molecular weight, those have same rate of diffusion. In the given mixture, CO, N_2 have molecular weight :44

66. $\Delta H_f = -68.39 - 48 - 14$

67.

As the graph of P vs $\frac{1}{V}$ would be a straight line.

a. is true because log P = log C – log V.

b. is true because $\frac{dP}{dV} = \frac{-C}{V^2}$

c. is true P = $\frac{C}{V}$ Where C is a constant.

$$\frac{V_1}{V_2} = \sqrt{\frac{M_2}{M_1}}$$

$$\Rightarrow \frac{50}{40} = \sqrt{\frac{M}{64}} \Rightarrow \frac{50}{40} = \frac{\sqrt{M}}{8}$$

$$\Rightarrow \frac{50 \times 8}{40} = \sqrt{M} \Rightarrow (10)^2 = M$$

69. $\Rightarrow M = 100$

70.

$$\frac{r_1}{r_2} = \sqrt{\frac{T_2}{T_1}}$$

$$2 = \sqrt{\frac{T_2}{273}}$$

$\therefore T_2 = 273 \times 4 = 1092$ K = 818°C

71. 8gm of oxygen means 0.25moles and 8gm of Helium means 2moles.
Molar ratio is 8 : 1.

72.

$\Delta U = 10$ J and $\Delta H = 990$ J

$\Delta U = \Delta Q + \Delta W$

for adiabatic process $\quad \Delta Q = O$

$\therefore \quad \Delta Q = \Delta W = -P\Delta V = -100 \times 10^5 (-10^{-6}) = 10$ J

$\Delta H = \Delta U + \Delta(PV)$

$\therefore \quad \Delta H = \Delta U + P_2V_2 - P_1V_1$

$= 10 + 100 \times 10^5 \times 99 \times 10^{-6} - 1 \times 10^5 \times 10 \times 10^{-6}$

$= 10 + 990 - 10 = 990$ J

73.

Given, $P_1 = P, V_1 = V, T_1 = T$

$P_2 = P_2, V_2 = V - \frac{100/21}{100}V, T_2 = T$; $\quad P \times V = P_2 \times \left(V - \frac{100/21}{100}V\right)$

$P_2 = \frac{21}{20}P$; $\quad \Delta P = \frac{1}{20}P$ $\quad$ % increase= $\frac{1}{20}\frac{P}{P} \times 100 = 5\%$ Ans.

74.

$C_2H_5 - C_2H_5 + S \longrightarrow C_2H_5 - S - S - C_2H_5$

$\Delta H_f^o(C_2H_5 - S - C_2H_5) = \Delta H_f^o(C_2H_5 - S - S - C_2H_5) - \Delta H_{s-s} + \Delta H_{sub}$ S.

$-201.9 = -147.2 - \Delta H_{s-s} + 222.8$

$\Delta H_{s-s} = 277.5$ kJ/mol.

75.

On mixing $CaCl_2(aq.)$ and Na_2CO_3

$CaCl_2 + Na_2CO_3 \longrightarrow CaCO_3 + 2NaCl$

Solutions are very dilute and thus, 100 % dissociation occurs.

$Ca^{2+}(aq.) + 2Cl^-(aq.) + 2Na^+(aq.) + CO_3^{2-}(aq.) \longrightarrow$

$CaCO_3 \downarrow \; + 2Na^+(aq.) + 2Cl^-(aq.)$ or

$Ca^{2+}(aq.) + CO_3^{2-}(aq.) \longrightarrow CaCO_3(s)$

$\Delta H = \Sigma H^0_{Products} - \Delta H^0_{Reactants}$ or $\Delta H = \Delta H^0_{f\,CaCO_3} - \left[\Delta H^0_{f\,Ca^{2+}} + \Delta H^0_{f\,CO_3^{2-}}\right]$

$\because \; H^0$ of a compound

$= H^0_{Formation} = -288.45 - (-129.80 - 161.65) = 3\,k\,cal$